Pathway U

Webwork HW#1

① $-5x_1 - 2x_2 - 17x_3 = -17$
$3x_1 + x_2 + 10x_3 = 11$

$x_2$ & $x_3$ are free variables
$x_2 = s_1$, $x_3 = s_2$

$3x_1 = 11 - s_1 - 10s_2$
$x_1 = \frac{11}{3} - \frac{s_1}{3} - \frac{10}{3}s_2$

$3x_1 = 11 - s_1 - 10s_2$

$-5\left(\frac{11}{3} - \frac{s_1}{3} - \frac{10}{3}s_2\right) - 2s_1 - 17s_2 = -17$
$-\frac{55}{3} + \frac{5s_1}{3} + \frac{50}{3}s_2 - 2s_1 - 17s_2 = -17$
$-55 + 5s_1 + 50s_2 - 6s_1 - 51s_2 = -51$
$-s_1 - s_2 = 54 \Rightarrow s_1 = -54 - s_2$

$3x_1 = 11 - (-54 - s_2) - 10s_2$
$3x_1 = 65 - 9s_2 \Rightarrow x_1 = \frac{65}{3} - 3s_2$
$x_2 = -54 - s_2$
$x_3 = s_2$

$-5(7 - 3s_1) - 2(1 + 2s_1) - 17s_1 = -17$
$-35 + 25s_1 - 2 - 4s_1 - 17s_1 = -17$
$25s_1 - 4s_1 - 17s_1 = -17 + 37$
$4s_1 = 20$
$s_1 = 5$

$-5(7 - 5s_1) - 2(1 + 2s_1) - 17(s_1) = -17$

② $2x_1 - x_2 - 2x_3 = 4$
$-3x_3 + 3x_4 = -1$

$$\left[\begin{array}{cccc|c} 2 & -1 & -2 & 0 & 4 \\ 0 & 0 & -3 & 3 & -1 \end{array}\right]$$

$x_1$ & $x_3$ = leading variables
$x_2$ & $x_4$ = free variables = $s_1$ & $s_2$

$-3x_3 = -1 - 3s_2$
$x_3 = \frac{1}{3} + s_2$

$2x_1 - s_1 - 2\left(\frac{1}{3} + s_2\right) = 4$
$2x_1 = 4 + s_1 + \frac{2}{3} + 2s_2$

$2x_1 = \frac{14}{3} + s_1 + 2s_2$
$x_1 = \frac{14}{6} + \frac{s_1}{2} + s_2$; $x_2 = s_1$; $x_3 = \frac{1}{3} + s_2$; $x_4 = s_2$

③ For what values of h & k does the linear system have ∞ solutions?

$6x_1 + 2x_2 = 3$
$hx_1 + kx_2 = 1$

$$\left[\begin{array}{cc|c} 6 & 2 & 3 \\ h & k & 1 \end{array}\right]$$

In order for a system to have ∞ solutions, the two rows must be linearly independent (scalar multiples of each other) $\Rightarrow h = \frac{1}{3}(6) = 2$, $k = \frac{1}{3}(2) = \frac{2}{3}$

④ For what values of h is the linear system consistent? (consistent = at least 1 soln)

$-3x_1 - 4x_2 = h$
$-9x_1 - 12x_2 = 1$

$$\left[\begin{array}{cc|c} -3 & -4 & h \\ -9 & -12 & 1 \end{array}\right]$$

Consistencies/inconsistencies can be discovered by turning it into reduced row echelon form

Reduced row echelon form = matrix in row echelon form, all pivots = 1 and pivots are the only non-zero entries in their columns

Row echelon form ⇒ same as reduced ref, except it can have other variables in the columns w/ the pivot points (1's)

How to convert to rref: ① interchange rows, so the row w/ first pivot colm is in $1^{st}$ row
② Multiply row, so pivot = 1
③ Add multiples of pivot row to lower rows, so every element in the pivot column of the lower rows = 0
④ Continue steps 1-3 on each row until in ref
⑤ Gauss-Jordan Elimination

$-\frac{1}{3}$

$$\left[\begin{array}{cc|c} 1 & \frac{4}{3} & -\frac{h}{3} \\ 0 & 0 & -\frac{9h}{3} + 1 \end{array}\right]$$

$9R1 + R_2$

$\frac{36}{3} + -(12)$

$-\frac{9h}{3} + 1 = 0$
$-\frac{9h}{3} = -1$
$\frac{9h}{3} = 1$ $3h = 1 \rightarrow h = \frac{1}{3}$

Webwork #1 continued

(5) $\begin{bmatrix} 5 & -3 & -1 & -3 \\ 0 & -5 & -1 & 2 \end{bmatrix} \Rightarrow \left.\begin{array}{l} 5x_1 - 3x_2 - x_3 = -3 \\ -5x_2 - x_3 = 2 \end{array}\right\}$ converted augmented matrix to the equivalent linear system

(6) Determine form of Matrix: $\begin{bmatrix} 1 & 7 & 0 & 2 & -8 \\ 0 & 0 & 1 & -8 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ } Reduced row echelon form (rref) because 1 is the leading variable & there are no other nonzero terms in the columns with pivots

(7) Determine form of Matrix: $\begin{bmatrix} 2 & 3 & 1 & 2 & 0 \\ 0 & 0 & -3 & 2 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix}$ } Not in echelon form because the leading term in E3 is not to the right of the leading term in row 2.

(8) $\left[\begin{array}{ccc|c} -2 & 7 & 4 & 3 \\ 1 & -3 & -1 & 1 \\ -5 & 12 & 0 & -18 \end{array}\right] \Rightarrow$ converting to ref means you want -2, -3, & 0 to become 1, and you want 1, -5, & 12 to become 0

(1) Identify pivot for row 1 $\Rightarrow$ change to 1 by switching $E_1$ & $E_2$: $\left[\begin{array}{ccc|c} 1 & -3 & -1 & 1 \\ -2 & 7 & 4 & 3 \\ -5 & 12 & 0 & -18 \end{array}\right]$

(2) Make 1st column (except pivot) 0 using row operations $(2R_1 + R_2)$: $\left[\begin{array}{ccc|c} 1 & -3 & -1 & 1 \\ 0 & 1 & 2 & 5 \\ -5 & 12 & 0 & -18 \end{array}\right]$ $(5R_1 + R_3) = \left[\begin{array}{ccc|c} 1 & -3 & -1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & -3 & -5 & -13 \end{array}\right]$ $(3R_2 + R_3) = \left[\begin{array}{ccc|c} 1 & -3 & -1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & -3 \end{array}\right]$. This system is consistent because it has one unique solution (triangular... back substitution)

$x_3 = -3 \Rightarrow x_2 + 2(-3) = 5 \Rightarrow x_2 = 11 \Rightarrow x_1 - 3(11) - (-3) = 1 \Rightarrow x_1 = 1 - 33 + 6 \Rightarrow -26$

$(3R_2 + R_1) = \left[\begin{array}{ccc|c} 1 & -2 & -1 & 6 \\ 1 & -3 & -1 & 1 \\ -5 & 12 & 0 & -18 \end{array}\right]$ $(-R_1 + R_2) = \left[\begin{array}{ccc|c} 1 & -2 & -1 & 6 \\ 0 & -1 & 0 & -5 \\ -5 & 12 & 0 & -18 \end{array}\right]$ $(-R_2) = \left[\begin{array}{ccc|c} 1 & -2 & -1 & 6 \\ 0 & 1 & 0 & 5 \\ -5 & 12 & 0 & -18 \end{array}\right]$

$(5R_1 + R_3) = \left[\begin{array}{ccc|c} 1 & -2 & -1 & 6 \\ 0 & 1 & 0 & 5 \\ 0 & 2 & -5 & 12 \end{array}\right]$ $2R_2 = \left[\begin{array}{ccc|c} 1 & -2 & -1 & 6 \\ 0 & 2 & 0 & 10 \\ 0 & 2 & -5 & 12 \end{array}\right]$ $-R_2 + R_3 = \left[\begin{array}{ccc|c} 1 & -2 & -1 & 6 \\ 0 & 2 & 0 & 10 \\ 0 & 0 & -5 & 2 \end{array}\right]$

(9) $\left[\begin{array}{ccc|c} -2 & 1 & -7 & -13 \\ -1 & 1 & -6 & -9 \end{array}\right]$